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ABSTRACT

Problem Solving in Mathematics (PSM) is a program of problem-solving lessons for grades 4-9. The content objectives of these lessons are similar to those of most textbooks. However, PSM emphasizes problem-solving skills and a variety of instructional strategies which foster the development of such skills. These strategies include direct instruction, guided discovery, laboratory work, small-group discussions, nondirective instruction, and individual work. This guide shows how PSM lessons should be used and how PSM activities can be integrated into the regular mathematics curriculum to provide students with a more stimulating problem-solving experience. Areas addressed include: the nature of problem-solving in mathematics; the philosophy of the PSM program; how PSM is different from other problem-solving materials; implementing PSM in schools; what research says about problem-solving; and evaluation of student work. Fifteen problems (with answers) for teachers to complete are included. These problems are designed to increase the teacher's confidence level and ability in problem-solving methods. Also included are suggestions for creating a classroom climate for problem-solving, a list of teacher behaviors indicative of problem-solving (in the form of a self-evaluation checklist), and an observational chart (with instructions) for evaluating problem-solving behaviors in the classroom. (JN)

Lane County Mathematics Project

PROBLEM SOLVING IN MATHEMATICS

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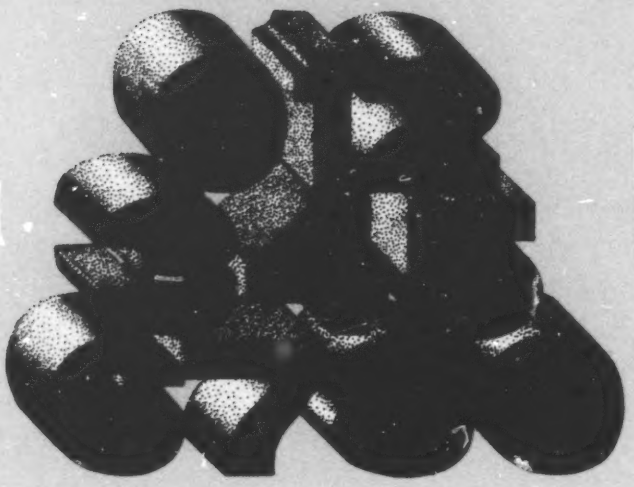
IN-SERVICE GUIDE



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Farmer McDonald raises ducks and cows.

The animals have a total of 9 heads and 26 feet.

How many ducks and how many cows does Mr. McDonald have?

As the pupils entered Miss Adams' classroom they found this "Ducks and "Cows challenge problem on the chalkboard. Miss Adams suggested they get into their groups of four and begin working. It didn't take them long to get started. They always enjoyed time spent on challenge problems.

After a while Miss Adams moved around the room to see what kind of solution strategies were being used in the different groups. Let's listen to some of the discussion.

The first group is using a guess, check, and refine strategy.

Sarah: I tried 2 ducks and 7 cows, but that gave a total of 32 feet--too many.

Mirko: We want fewer feet, so we need to cut down on the number of cows.

Alan: But we're pretty close. So don't cut it down too far.

Another group decided to make a systematic list of possible results.

Rosa. Let's make a list. Try 0 ducks and 9 cows; then 1 duck and 8 cows, and so forth.

Ducks	Cows	Total Heads	Total Feet
0	9	9	36
1	8	9	34
2	7	9	32

Jeff: I see a pattern showing up in the feet column. We should be able to solve the problem by extending this pattern.

The most interesting strategy Miss Adams observed was one in which a drawing was being used. Eric explained it this way:

> Put 9 dots on your paper. These are the 9 heads. Under each dot, put 2 lines. This gives us 18 feet. But we need more feet, so add 2 more lines on some of them until we have 26.

The next group had already solved the problem so Miss Adams suggested they try to create other Ducks And Cows problems, some that are possible to solve; and also, some that have no solution. The problems that this group created were quite interesting. One is shown below.

Sally: How many ducks and cows are there if there are 6 heads and 17 feet?

Jose: Are we allowed to have one-legged ducks or three-legged cows?

* * * * * * * * * * * * * * * * * *

Welcome to PROBLEM SOLVING IN MATHEMATICS. The above classroom episode was created to give you a feeling for the kind of problem-solving atmosphere that can permeate a classroom. The teacher is available to answer questions, to give hints when necessary, and to provide extensions. But generally speaking, the groups are left pretty much on their own to come up with their own solution strategies in a cooperative way.

It's important to emphasize that pupils were using several different problem-solving skills to reach a solution:

- Guess and check.
- Look for a pattern.
- Make a systematic list.
- Make a drawing.

These and other problem-solving skills need to be taught early in the school year. Other problem-solving activities like "Ducks And Cows" should be presented only after the Getting Started section has been completed and pupils have had some successful problem solving experiences.

Of course, PROBLEM SOLVING IN MATHEMATICS contains much more than Getting Started activities and challenge problems. There are drill and practice sections, many laboratory activities, and actual content sections covering such topics as percent, geometry, fractions, and decimals.

The purpose of this guide is to show how the material should be used and how the activities can be integrated into your regular mathematics curriculum to provide pupils a more stimulating problem solving experience.

SOME QUESTIONS AND ANSWERS

WHAT IS PROBLEM SOLVING IN MATHEMATICS?

PROBLEM SOLVING IN MATHEMATICS (PSM) consists of six books for grades 4 to 8 and algebra. There are approximately 80 lessons per grade level along with teacher commentary. PSM lessons cover, to some extent, most of the content studied at each grade level. An alternative book for low achievers in grades 4-6 is also available.

IS PROBLEM SOLVING IN MATHEMATICS A COMPLETE CURRICULUM?

PSM is not intended to be a complete mathematics program. Neither is it to be considered supplementary in the sense of being extra credit or to be done on special days. It was designed to be integrated into the regular mathematics program for each grade level. Lessons selected from PSM frequently can be used to replace textbook pages.

HOW IS PSM DIFFERENT FROM OTHER PROBLEM-SOLVING MATERIALS?

PSM provides more than just non-routine puzzle-type problems. Also, PSM provides more than just story problems. In addition to these, many problem-solving activities are included that fit a usual curriculum--topics like whole numbers, fractions, decimals, percents, and equation solving.

From 7th grade Drill And Practice

GOING IN CIRCLES

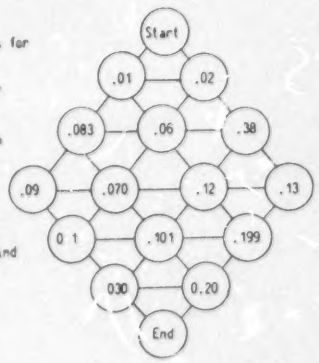
Rules

. Don't count the circles for "start" and "end."

. Always move to a larger number.

1. Find the longest path through the maze.

2. There are four different paths that are the shortest. Find all four of them.



Find a path that is exactly six circles long.

WILL PSM MAKE IT EASIER TO TEACH MATHEMATICS?

The content units (fractions, decimals, percents, etc.) should make it easier for both teachers and pupils. A visual hands-on approach often is used to develop and extend difficult concepts. An intuitive understanding of mathematics is the primary objective. Observation has shown that pupils enjoy learning from the PSM materials. This in itself often makes teaching mathematics easier.

HOW CAN A TEACHER LEARN HOW TO USE PSM?

The best way is to attend some PSM inservice sessions. But these may not be available in your region. The next best way is to spend time listening to the self-inservicing audio tapes. These are

- Teacher Interview/Principal Interview
- Getting Started/Challenge Problems (one for grades 4,5,6 and one for grades 7,8,algebra)
- Overview/A Closer Look at Content Sections (one for each grade level 4,5,6,7,8, and algebra)

Additional teaching suggestions are included in the Teacher Commentary found on the back of each lesson. Each section within a book also has an overview teacher commentary.

HOW CAN A TEACHER BECOME MORE CONFIDENT IN PROBLEM SOLVING?

In order to teach problem solving effectively, teachers must have confidence in their own problem-solving ability. A teacher develops this confidence in the same way pupils do--by have successful problem-solving experiences. Section VII of this guide has material designed to assist you with this extra "inservice."

"Solving problems is a practical art, like swimming, or skiing, or playing the piano; you can learn it only by imitation and practice."

George Polya

HOW MUCH TEACHER TIME IS REQUIRED?

Initially, PROBLEM SOLVING IN MATHEMATICS requires additional teacher time. During the second year, however, much less time is needed for preparation. Some items that take extra time are listed below.

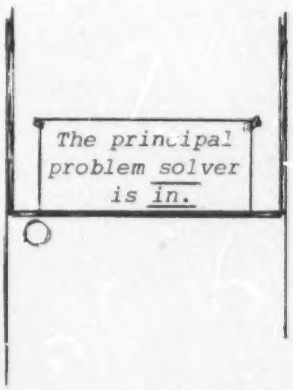
- Understand the philosophy of the program and how and when the materials should be used.
- Attend PSM inservice sessions and/or listen to the self-inservicing audio tapes.
- Study the teacher commentary.
- Work through the lessons and problems before giving them to pupils.

WHY IS IT BEST TO HAVE WHOLE-STAFF COMMITMENT?

Improving pupils' abilities to solve problems is not a short-range goal. In general, efforts must be made over a long period of time if permanent changes are to result. Ideally, then, the teaching staff for at least three successive grade levels should commit themselves to use PSM with their pupils. Also, if others are involved, this will allow for opportunities to plan together and to share experiences.

HOW IMPORTANT IS THE SUPPORT OF THE PRINCIPAL?

The principal is quite often the curriculum director of the school. Thus, her or his support for curriculum programs like PSM is vital in order for the program to succeed. A special audio tape has been prepared for principals. This tape is designed to acquaint them with PSM and the kinds of administrative responsibilities and support necessary.



WHAT DOES RESEARCH SAY ABOUT PROBLEM SOLVING?

An article, "Untangling Clues from Research in Problem Solving," by Marilyn N. Suydam, in PROBLEM SOLVING IN SCHOOL MATHEMATICS, the 1980 Yearbook of the National Council of Teachers of Mathematics, summarized the recent research into mathematical problem solving. Several excerpts pertinent to the Problem Solving in Mathematics Project are listed here. The author and year are included below each excerpt.

> Experience and some research suggest that certain problem-solving skills which will improve mathematical problem-solving performance can be learned--provided the teacher illustrates how the procedure works, gives ample opportunity for discussion, practice, and reflection; and supports and encourages the learner's efforts.
> (Kilpatrick, 1978)

> Training in a variety of problem-solving skills is necessary so that students have a repertoire from which they can draw as they meet the wide variety of problems that exist.
> (Brandau and Dorsey, 1979)

> When problem-solving skills are specifically taught, they are then used more and students achieve correct solutions more frequently.
> (Clement, et al., 1977)

> Allow children sufficient time to solve problems. Not all problems can be solved within the confines of one mathematics lesson; some need to be 'mulled over' for longer period of time.
> (Suydam, 1980)

> ...practice in problem solving should not consist in repeated experiences in solving the same problems with the same techniques, but should consist of the solution of different problems by the same techniques and the application of different techniques to the same problems.
> (Brownell, 1942)

> Manipulative materials, pictures, diagrams, and similar aids enhance the probability of a problem being understood and solve correctly.
> (Ammon, et al., 1973)

> Problem-solving skills are improved by incorporating them throughout the curriculum--...
> (Roman, 1975)

> Do not be misled by the immediate benefits of structural [expository] teaching--the long-range benefits of teaching through problem solving may be higher.
> (Flener, 1978)

> Instead of being 'protected' from error, the child should many times be exposed to error and be encouraged to detect and to demonstrate what is wrong, and why.
> (Brownell, 1942)

EVALUATION OF STUDENT WORK

To grade or not to grade is often a question asked about problem solving. If, indeed, the goal is to get pupils to take risks, to persevere, to concentrate on process, should some kind of a grade be placed on their work? A most common response is, yes, we should, for several reasons. First, pupils often perceive that if a topic isn't being graded, then it isn't important. This is definitely not the perception pupils should have. Second, if you follow the suggestions offered you will be spending about one-third of your instruction time teaching mathematics through a problem-solving approach. This amount of time commitment certainly needs a process to identify whether pupils are "achieving" in problem solving.

The word grade was purposely used above for emphasis. A far better word is evaluate. The two are not synonymous. Some teachers feel that no grades at all should be given while others are in situations where grades must be given. Whatever the situation, you should, at least, consider some of the suggestions below.

In problem solving whatever evaluation is done needs to focus on the process and effort rather than the final answer. Is there evidence of willingness to try a problem? Does a pupils display perseverance when the solution is not immediately obvious? Can a pupil pick out an appropriate problem-solving skill? Is the plan of attack carried out? Does the pupil look back and reflect on the outcome? Can the knowledge gained from solving this problem transfer to another problem? Several schemes of assessing pupil work are listed below. You need to use any or all of them, depending upon your individual feelings. No priority is intended in the listing.

A. Written/Verbal Comments

Collect papers from pupils. Read and comment according to the questions mentioned above. Activities done as group activities may best be done with verbal comments to the group as a whole. This type of assessment offers you the opportunity to mention other skills that might have been used to solve the problem.

B. +, ✓, -

This is similar to the scheme often used to assess homework papers. A plus means better than average effort/achievement. A check indicates average, and a minus shows less than average effort/achievement.

C. A,B,C,D,E

This is the traditional scheme of assigning a letter grade to the effort/achievement of a pupil. Remember, the criteria for a grade needs to reflect the questions mentioned above, not merely whether the pupil had the correct answer.

D. 0,1,2

This point system was developed as part of a Title IV-C project entitled Mathematical Problem Solving, Randall Charles and Robert Mason, Directors. Notice that it too relies on much more than just getting the correct answer. You might decide on one score of 0, 1, or 2 or the sum of three scores giving a final point total of anywhere from 0 to 6.

Understand the problem:	0 - completely misinterprets the problem 1 - misinterprets part of the problem 2 - complete understanding of the problem
Solving the problem:	0 - no attempt or a totally inappropriate plan 1 - partly correct procedure based on part of the problem interpreted correctly 2 - a plan that could lead to a correct solution with no arithmetic errors
Answering the problem:	0 - no answer or wrong answer based on an inappropriate plan 1 - copying error; computational error; partial answer for problem with multiple answers; wrongly-labeled answer 2 - correct solution

Another way of assessing achievement in problem solving is to include problem solving items on your quizzes. It is not a difficult task to create items that might assess whether pupils can (a) recognize a problem-solving skill already used, (b) pick a skill to be used, (c) pick an alternative skill to use, (d) actually use a skill to solve a problem, or (e) understand a problem. An example of each follows.

(a) Name the problem-solving skill used to solve this problem.

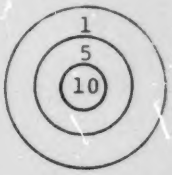
A pencil costs 7¢. Find the cost of 6 pencils.

Solution:

Pencils	Cost
1	7¢
2	14¢
3	21¢
4	28¢
5	35¢
6	42¢

(b) Name a problem-solving skill you would use to solve this problem.

Tracy throws three darts at the target.
All darts hit the target.
What are the possible scores Tracy might get?



(c) One way to solve this problem is shown. Name a different problem-solving skill that could be used.

The Brooks family and the Avezanno family want to combine their ducks and beavers. Altogether they will have animals with 10 heads and 26 feet. How many ducks and how many beavers do they have?

One solution: Guess and check:

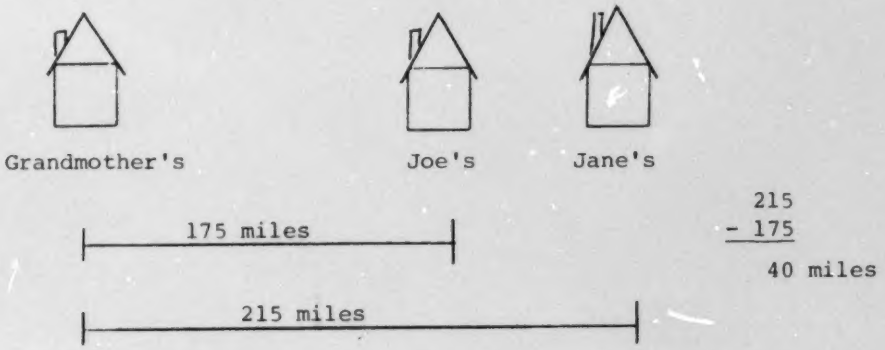
6 ducks and 4 beavers
12 feet 16 feet
28 feet in all
too many feet

7 ducks and 3 beavers
14 feet 12 feet
26 feet in all
just right

(d) Discover the pattern and write the next 3 numbers.

1, 1, 3, 6, 6, 8, 11, 11, 13, 16, 16, 18, 21, ___, ___, ___

(e) Write a word problem to match this answer.



One teacher using the PSM materials actually constructed an entire quiz on problem solving. An edited copy of that quiz follows. Notice how she has incorporated parts of (b) and (d) from the previous examples.

Quiz

Name _____ Date _____ Period _____

Look below at the list of problem solving skills we have studied so far.

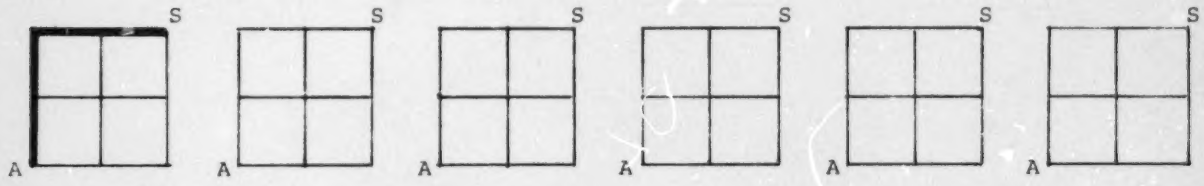
Skill Name:

a. Make a systematic list.
b. Make or use a drawing.
c. Look for a pattern.
d. Guess and check.

Directions: Read each of the following problems carefully. Choose a letter from the list above that names the skill you think you will use. You may name the skill you used after you do the problem.

1. Ann lives at A. There are six different routes she can use to walk to school. Can you find them all? The first one is done for you.

Skill _____



2. In each chart, find the rule that will help you find the second number if you know the first. Fill in the answers.

Skill _____

a.

8	3
15	10
30	25
50	
25	
	100

b.

5	20
6	24
10	40
8	
	100
	60

c.

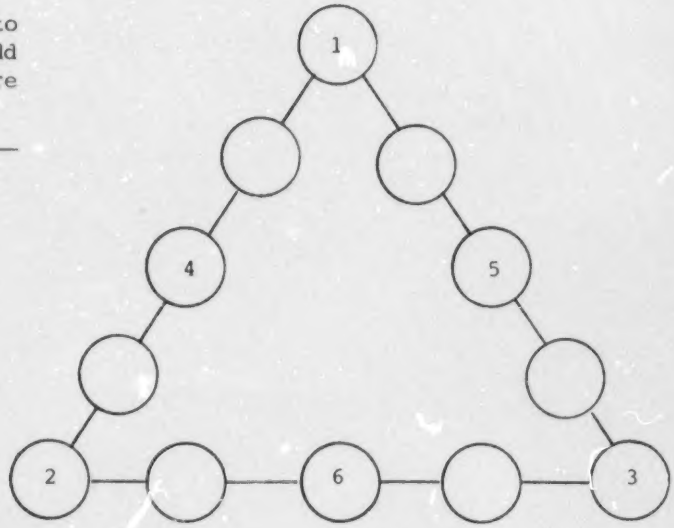
1	1
2	4
3	9
4	
	100
	81

d.

2	7
3	10
5	16
8	
22	
	22

3. Use each of the numbers 1 to 12 to make each side of the triangle add to the same sum. Some numbers are already in the correct circles. What is the "magic" sum? _____

Skill ____



4. There are 6 basketball teams in a tournament. The teams are lettered A through F. Each team plays each of the other teams twice. How many games are played altogether? _____

Skill _____

17

PROBLEMS TO TRY

The primary purpose of this set of problems is to increase your own confidence level and ability in problem solving. In order to accomplish this objective, certain steps MUST be followed.

- Use a piece of paper to cover up everything except the statement of the problem.
- Try to solve the problem without relying on any hints. If you become frustrated, move your paper down to uncover the first part of the solution path. This should help you get started.
- If you are still frustrated after several minutes of work, move your paper down to uncover the next part of the solution path.
- Continue this process until you reach a solution.

Hopefully, you'll soon be able to solve many of the problems without uncovering all of the hints. Nevertheless, you should still compare your solution path with ours. This comparison likely will focus on certain patterns which may be beneficial later on.

After you have completed the problem, you then are asked to create your own variation of the problem and write up the solution path which shows the process used in finding the answer. Do not neglect this. To be able to create and solve new problems is an extremely important step in becoming a confident problem solver.

Finally, the sequence of the problems is important. You should work them in the order given and not skip around.

PROBLEM 1

The faculty room was alive. The teachers were eager to greet one another and to begin teaching PROBLEM SOLVING IN MATHEMATICS.

Everyone in the room made a point to shake hands with everyone else. How many persons were in the room if there were a total of 78 handshakes?

* *

. LOOK AT SIMPLER CASES.

How many handshakes for 2 persons? for 3 persons? for 4 persons?

. ORGANIZE THE RESULTS IN A TABLE.

Persons	2	3	4	5	...
Handshakes	1	3	6		

. LOOK FOR A PATTERN.

Persons	2	3	4	5	6	7	8	9	...
Handshakes	1	3	6	10	15	21			

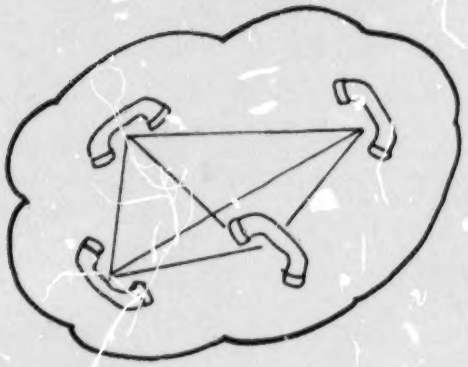
Notice the pattern: 1, 3, 6, 10, 15, 21, ...
Each time, the increase is one more than the previous one.

ANSWER: Thirteen persons were in the room.

* *

Vary the problem. Be sure to write up your solution path which shows the steps used in finding the answer. For example, one variation is the handshake problem on page 6. Another might be to ask the question, "How many more handshakes would there be if two more persons came into the faculty room?" A third variation might be to change the context like that shown in problem 2 on the next page.

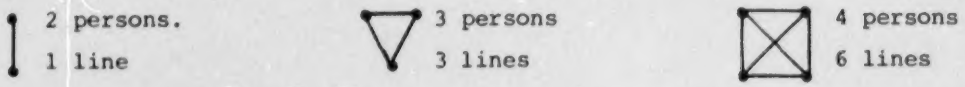
PROBLEM 2



Fifteen talkative teenagers decided to create their own telephone system. Telephone lines were set up so that every person could call each of the other 14. How many lines are needed altogether?

* *

. LOOK AT SIMPLER CASES.



. ORGANIZE THE RESULTS IN A TABLE.

Persons	2	3	4	5	...
Telephone lines	1	3	6		

. LOOK FOR A PATTERN.

Persons	2	3	4	5	6	7	8	9	...
Telephone lines	1	3	6	10	15	21			

Notice the pattern: 1, 3, 6, 10, 15, 21, ...
Each time the increase is one more than the previous one.

ANSWER: 105 telephone lines

* *

Vary the problem. Be sure to write up your solution path.

PROBLEM 3

Most pizzas are cut through the center like this. Notice that 5 cuts are shown, resulting in 10 pieces. Bill claims that he can get 16 pieces by still using only 5 cuts. Show how this is possible. (The cuts must be straight and must go from edge to edge.)

* *

. MAKE SIMPLER DRAWINGS .

Find the maximum number of pieces.

no cuts;
1 piece

one cut;
2 pieces

two cuts;
4 pieces

three cuts;

(Find the maximum number of pieces. Remember, the cuts do not have to go through the same point.

. ORGANIZE THE RESULTS IN A TABLE .

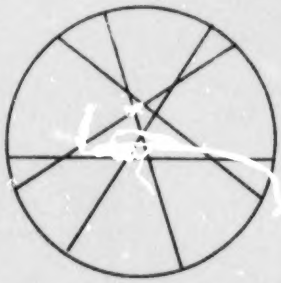
Number of cuts	0	1	2	3	...
Number of pieces	1	2	4	7	

. LOOK FOR A PATTERN.

Number of cuts	0	1	2	3	4	5
Number of pieces	1	2	4	7	11	16

Notice the pattern: 1, 2, 4, 7, 11, 16.
Each time, the increase is one more than the previous one.

ANSWER:



* *

Vary the problem. Be sure to write up your solution path.

DISCUSSION TIME

It's quite possible that you solved the first three problems using a different strategy than the one we presented. Perhaps your strategy is even more efficient. Still, it's important that you recognize the skills we've used. The important ones are underlined in each solution path.

Another skill you probably used is one commonly called "looking back." You undoubtedly noticed that the first three problems all used a similar strategy. Thus, by looking back at the solution for problem 1, the other two problems could be solved in a similar manner. This is why it's essential to verbalize skills and solution paths with pupils. This may be helpful in analyzing and solving problems occurring later.

Several additional skills will be emphasized as you work through the remaining problems in the set. They are:

. Guess and check.

. Use a systematic list.

. Make a drawing.

. Eliminate possibilities.

Remember to continue with your endeavors to vary the problems. You may even want to try writing some extensions. Good extensions are important to have for those pupils who need a further challenge.

> *"Good problems and mushrooms of certain kinds have something in common; they grow in clusters. Having found one, you should look around; there is a good chance that there are some more quite near."*
>
> George Polya

PROBLEM 4

As a promotional gimmick, the owner of the local cycle shop made up an interesting puzzle. Give it a try. See if you would have won a prize.

SOLVE THIS PROBLEM
AND
WIN A PRIZE

This store sells only bikes and trikes. Altogether, there are 23 seats and 50 wheels.

How many bikes and how many trikes are there?

* *

. MAKE A GUESS.

Show that a guess of 10 bikes and 13 trikes is too small.

. USE A CHART. REFINE YOUR GUESS.

Bikes		Trikes		Total	Total
Seats	Wheels	Seats	Wheels	Seats	Wheels
10	20	13	39	23	59
14	28	9	27	23	55
20	40	3	9	23	49
19	38	4	12	23	50

ANSWER: 19 bikes and 4 trikes

* *

Vary the problem. Be sure to write up your solution path.

PROBLEM 5

Sam bought 24 pencils. Some cost 15¢ each; some cost 20¢ each. How many of each kind did he buy if he paid $4.00 for the pencils?

* *

. MAKE A GUESS.

Show that a guess of 10 fifteen-cent pencils is too large.

. USE A CHART. REFINE YOUR GUESSES.

15¢ Pencils		20¢ Pencils		Total
Number	Cost	Number	Cost	Cost
10	$1.50	14	$2.80	$4.30
12	1.80	12	2.40	4.20
14	2.10	10	2.00	4.10
16	2.40	8	1.60	4.00

ANSWER: 16 fifteen-cent pencils and 8 twenty-cent pencils.

* *

Vary the problem. Be sure to write up your solution path.

PROBLEM 6

Eric has some quarters and nickels.
He has 8 more nickels than quarters.
The total value is $3.70.
How many of each coin does Eric have?

* *

. MAKE SOME GUESSES AND CHECK THEM OUT.

. USE A CHART. REFINE YOUR GUESSES.

Quarters	Cents in Quarters	Nickels	Cents in Nickels	Total amount in cents
5	125	13	65	190
10	250	18	90	340
11	275	19	95	370

ANSWER: 11 quarters and 19 nickels

* *

Vary the problem. Be sure to write up your solution path.

PROBLEM 7

Mr. Kannen is a very fussy photographer. He has already lined up his four children in several different ways. He is still not satisfied. How many different arrangements are possible?

* *

. USE A SYSTEMATIC LIST.

Use letters to represent the children.

ABCD	BACD	
ABDC	·	
ACBD	·	
ACDB	·	. . .
ADBC	·	
ADCB	·	

. LOOK FOR PATTERNS.

There are 6 different arrangements with A in the first position; there are 6 different arrangements with B in the first position; etc.

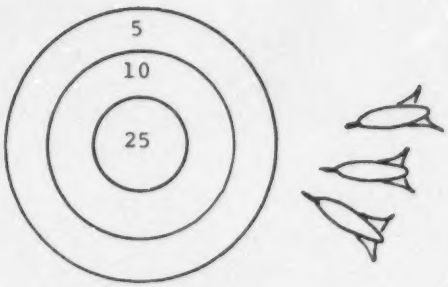
ANSWER: 24 different arrangements

* *

Vary the problem. Be sure to write up your solution path.

PROBLEM 8

Joan is setting up a dart contest. Each person will throw 3 darts. If all 3 darts hit the target, how many different scores are possible?



* *

. USE A SYSTEMATIC LIST.

25	10	5	Total
✓✓✓			75
✓✓	✓		60
✓✓		✓	55
✓	✓✓		45
✓	✓	✓	40
✓		✓✓	35
	✓✓✓		30
	✓✓	✓	25
	✓	✓✓	20
		✓✓✓	15

ANSWER: 10 different scores are possible.

* *

Vary the problem. Be sure to write up your solution path.

PROBLEM 9

While waiting for their dinner to arrive, the Smiths decided to determine how many different three-course meals could be ordered.

1st course -- soup or salad
2nd course -- chicken, beef, lamb, or ham
3rd course -- pie, cake, or sherbert

How large would the Smith family have to be for each of them to have a different meal?

* *

. USE A SYSTEMATIC LIST.

Soup-Chicken-Pie	Soup-Beef-Pie	Soup-Lamb-Pie	Soup-Ham-Pie
Soup-Chicken-Cake	Soup-Beef-Cake	Soup-Lamb-Cake	Soup-Ham-Cake
Soup-Chicken-Sherbert	Soup-Beef-Sherbert	Soup-Lamb-Sherbert	Soup-Ham-Sherbert

The above list shows 12 combinations. Twelve more can be found by replacing "soup" with "salad."

ANSWER: The Smiths would have to have a family of 24.

* *

Vary the problem. Be sure to write up your solution path.

PROBLEM 10

Each subtraction problem at the right uses the digits from 0 to 7. Rearrange the digits so the result is less than 250.

7531	7431	7042
- 6420	- 6520	- 6531
1111	911	511

* *

. LOOK BACK at the examples shown.

How was each of them adjusted to get a smaller answer?

. MAKE SOME GUESSES.

Try other numbers in the thousands position, like 2 and 1.

. REFINE YOUR GUESSES by looking back at what you've done.

ANSWER: The smallest answer is 247.

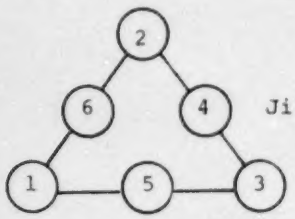
4012
- 3765
247

* *

Vary the problem. Be sure to write up your solution path.

PROBLEM 11

The puzzle at the right was solved differently by Jill and Joe. Jill found a solution with a magic sum of 9.

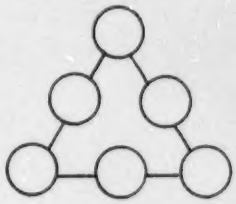


Jill's solution

Sum of 9

Use the numbers 1 to 6.

Each row of circles must have the same sum.



Joe's magic sum was 12. Show how Joe must have placed the numbers in his solution. Are there solutions with magic sums of 10 and 11? If so, find them.

* *

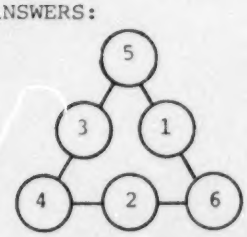
. LOOK BACK at Jill's solution. Study the placement of the largest numbers.

. MAKE SOME GUESSES.

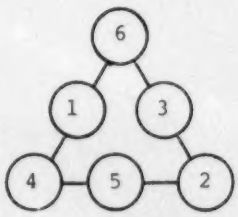
Try the largest numbers in different positions.

. REFINE YOUR GUESSES by looking back at what you've done.

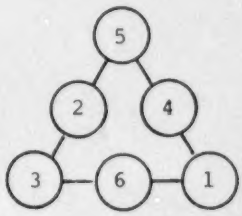
ANSWERS:



Sum of 12



Sum of 11



Sum of 10

* *

Vary the problem. Be sure to write up your solution path.

PROBLEM 12

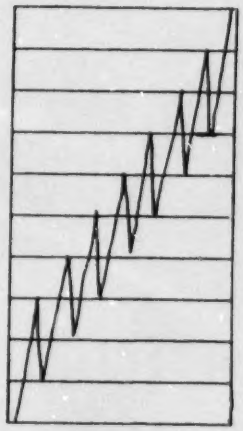
A frantic frog is at the bottom of a 10-foot well. Each hour he jumps up 3 feet, but then loses his "grip" and slides back down 2 feet. If he continues this maneuver, in how many hours will he be out of the well?

* *

. USE A DRAWING.

Keep track of his progress hour by hour.

ANSWER: 8 hours. The drawing at the right shows how this is possible.



* *

Vary the problem. Be sure to write up your solution path.

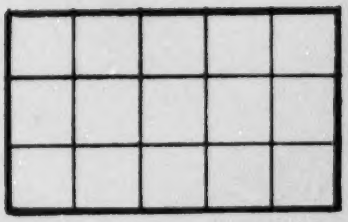
PROBLEM 13

Window panes come in three types:

Corner panes

Edge panes

Center panes



The 3 by 5 window at the right uses 4 corner panes, 8 edge panes, and 3 center panes. How many of each type of pane is used in a

3 by 6 window? 3 by 10 window? 3 by 100 window?

* *

. USE A DRAWING.

The drawing above suggests that for a 3 by 6 window two more edge panes are needed and one more center pane is needed.

. MAKE A TABLE. LOOK FOR PATTERNS. MAKE GENERALIZATIONS.

	3 by 5	3 by 6	3 by 7	3 by 8	...
Corner	4	4	4		
Edge	8	10	12		
Center	3	4	5		

ANSWERS; 3 by 6 -- 4 corner; 10 edge; 4 center
3 by 10 -- 4 corner; 18 edge; 8 center
3 by 100 -- 4 corner; 198 edge; 98 center

* *

Vary the problem. Be sure to write up your solution path.

PROBLEM 14

How old is Sam? Here are some clues:

- Sam is less than 75 years old.
- His age is an odd number.
- The sum of the digits of his age is 8.
- His age is a prime number.
- Sam has 3 great-grandchildren.

Which clue is NOT needed to find the answer?

* *

. ELIMINATE POSSIBILITIES.

The clue that eliminates the most possibilities is the third one. The remaining candidates are: 8, 17, 26, 35, 44, 53, 62, 71, and 80.

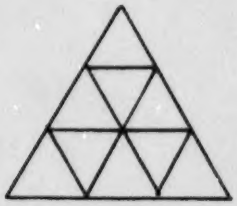
. ELIMINATE POSSIBILITIES by working with the other clues.

ANSWER: 71 years old. (The clue that is unnecessary is the second one. All primes, except for 2, are odd numbers.)

* *

Create your own problem, giving just enough hints to allow someone else to solve it. Be sure to write up your solution path.

PROBLEM 15



This triangular grid is 3 units on each side. It contains 9 small triangles. How many small triangle are in a triangular grid 20 units on a side.

Note: No hints are given for this problem, but several solution paths are shown. Before studying them, see how many different paths you can find.

Draw a figure with 20 units on a side and count the small triangles. The result is 400. (A tedious method.)

****

Start with simple cases and look for patterns.

Units on a side	1	2	3	4	5	...
Small triangles	1	4	9	16	25	

The sequence reveals a square number pattern. The result is 20^2 or 400.

****

Count the number of triangles in each row starting from the top.

Row	1	2	3	4	5	...	19	20
Small triangles	1	3	5	7	9		37	39

The solution is the sum of the first 20 odd numbers. This can be done in various ways. An easy way is to recognize that the sum of the first and last is 40, the second and second to last is 40, etc. There are 10 sums of 40. The result is 10 x 40 or 400.

****

Start with simpler cases. First count those triangles pointing upward. Then count those pointing downward. Look for patterns.

Units on a side	1	2	3	4	5	6
Triangles pointing up	1	3	6	10	15	?
Triangles pointing down	0	1	3	6	10	?

Notice the pattern in the bottom two rows. The difference becomes one greater as you move across the table from one entry to the next. This sequence is sometimes called the triangular number pattern. To solve the challenge for 20 units on a side, continue the pattern. Add the 20th triangular number to the 19th triangular number. The result is 210 + 190 or 400.

CLASSROOM CLIMATE

"A teacher who models the same behaviors desired in students can create a classroom learning condition in which maximum student growth will most likely occur."

Arthur L. Costa

SPECIFIC SUGGESTIONS

. Set an example by solving problems and by sharing these experiences with the pupils.

. Reduce anxiety by encouraging communication and cooperation. On frequent occasions problems might be investigated using a cooperative mode of instruction along with brainstorming sessions. (For an example of a successful grouping structure for problem solving, see "Groups of Four: Solving the Management Problem," by Marilyn Burns, Learning, 1981.)

. Encourage pupils in their efforts to solve a problem by indicating that their strategies are worth trying and by providing them with sufficient time to investigate the problem. Stress the value of the procedures pupils use.

. Use pupils' ideas (including their mistakes) in solving problems and developing lessons.

. Ask probing questions which make use of words and phrases such as

I wonder if -
Do you suppose that -
What happens if -
How could we find out -
Is it possible that -

. Encourage pupils to ask themselves probing questions. Pupils seldom are skilled at seeking probing questions, but they can be taught to do so. If instruction is successful, questions of the type, "What should I do now?," will be addressed to themselves rather than to the teacher.

. Make the physical environment of the classroom reflect problem solving.

- Post a list of problem-solving skills.
- Create bulletin boards on problem solving.
- Provide extra credit problem solving sheets.
- Display problem-solving resource and puzzle materials.
- Develop problem-solving activity stations.
- Display problem-solving posters.
- Run a problem-of-the-week contest.

SELF EVALUATION

The traits below are commonly found in classrooms with a good problem-solving climate. For each item, rate yourself as High, Medium, or Low. You be the judge as to which items need improvement. Do this now and again in the spring. Have you made improvements?

Accepts student ideas	H	M	L
Develops student ideas	H	M	L
Provides alternatives	H	M	L
Encourages perseverance	H	M	L
Provides challenges	H	M	L
Conducts brainstorming sessions	H	M	L
Encourages cooperation	H	M	L
Provides hints	H	M	L
Encourages independent thinking	H	M	L
Encourages questioning	H	M	L
Asks probing questions	H	M	L
Reviews problem-solving skills	H	M	L
Encourages strategy development	H	M	L
Adapts PSM material to own class needs	H	M	L

OBSERVATION CHART

Teacher _____ School _____ Grade Level _____

Title of Activity _____ Date _____ Time _____

ACTIVITY				ORGANIZATION						BEHAVIOR									
INTRO	PS	STRAT	UNREL	L_T	L_S	L_{SD}	S	I	L	1	2	3	4	5	6	7	8	9	10

Class Activities

INTRO - Introduction of problem-solving activity

PS - Student involvement in problem-solving activity

STRAT - Discussion of problem-solving strategies or skills

UNREL - Activity that is not directly related to problem solving

Classroom Organization

L_T - Large group; teacher directed (Lecture)

L_S - Large group; student centered (Discussion)

L_{SD} - Large group; student directed

S - Small group; cooperative learning

I - Individual involvement

L - Laboratory activity

Teaching Behaviors

1. Encourages development of strategies.
2. Accepts and develops students' ideas.
3. Reviews problem-solving skills.
4. Encourages perseverance.
5. Encourages cooperation.
6. Conducts brainstorming sessions.
7. Uses effective questioning strategies.
8. Provides challenges.
9. Provides alternatives.
10. Provides hints and cues.

Rev. 2/12/82

DIRECTIONS FOR USING THE OBSERVATION CHART

ACTIVITY SECTION

Enter the time that the first activity begins in the appropriate column of the first row. For example, if the teacher starts the class at 9:15 by having students discuss the strategies they used in solving a problem assigned the previous day, enter "9:15" in the "STRAT" column of the first row. Whenever a new activity is begun, the time should be recorded in the appropriate column of the next row.

ORGANIZATION SECTION

During any given activity, enter the numbers 1, 2, 3, etc., in the appropriate organization columns to indicate the sequence in which the different classroom organizations were used. For example, a teacher might use the following sequence of classroom organizations during the first activity mentioned above: 1. large-group-teacher directed, 2. small group, 3. large group-student centered, and 4. large group-teacher directed again. This would result in a "1" being recorded in the L_T column, a "2" in the S column, a "3" in the L_S column and a "4" after the "1" in the L_T column (all in the first row).

BEHAVIOR SECTION

Teaching behaviors are recorded in the same manner as were strategies. Enter a tally in the appropriate box every time a behavior is observed during a given activity.

TEACHING BEHAVIORS

1. Encourages development of strategies

   Demonstrated by: having students discuss strategies in small and large groups, having students map their strategies or solution paths, summarizing a variety of strategies that students have used to solve a given problem.

2. Accepts and develops students' ideas

   Demonstrated by: giving either verbal or non-verbal recognition of students' ideas, incorporating ideas into the day's lesson, expanding a student's idea, or suggesting further investigation of an idea or question.

3. Reviews problem-solving skills

   Demonstrated by: referring to problem-solving skills by name, asking students to identify the skills they have used or intend to use in solving a problem.

4. Encourages perseverance

   Demonstrated by: giving encouragement or positive reinforcement, suggesting a team effort, making adjustments in problem difficulty when necessary.

5. Encourages cooperation

   Demonstrated by: providing settings for students to work together in small groups in class and encouraging them to work cooperatively outside of class.

6. Conducts brainstorming sessions

   Demonstrated by: setting up small or large group discussions for the purpose of probing possible problem solution strategies.

7. Uses effective questioning strategies

   Demonstrated by: giving positive recognition of student questions, maintaining a non-threatening classroom atmosphere, proving a variety of problems that have multiple answers or no answer at all, asking probing questions and allowing adequate wait time between questions.

8. Provides challenges

   Demonstrated by: encouraging students to work on problem extensions and to write problems for others to solve, providing students with a variety of interesting, challenging problems.

9. Provides alternatives

   Demonstrated by: giving flexible assignments, allowing students to work individually or in groups, providing a variety of ways in which solutions may be demonstrated (paper and pencil, verbal presentation, etc.).

10. Provides hints and cues

    Demonstrated by: suggesting problem-solving skills to be considered, asking a question that may lead to a student idea, providing appropriate manipulatives, rephrasing a problem, suggesting the review of a related problem. (A cue is less directive than a hint.)

DIRECTIONS FOR WRITING THE OBSERVATION REPORT

A. Complete observation chart (see instructions accompanying the chart).

B. Make a subjective analysis of the extent to which a teacher is using the PSM materials in the way they are intended to be used. The following issues should be addressed:

1. Was the teacher prepared to teach the lesson?
2. What is the general classroom atmosphere?
3. Was the physical environment of the classroom conducive to problem solving?
4. To what extent do you <u>think</u> the teacher uses the PSM materials?
5. Do you feel the observed lesson represented a "normal" lesson?
6. Were the students "with" or "against" the teacher?
7. To what extent did the teacher read and comprehend the teachers' commentary?
8. Did the teacher adapt the materials in some way so as to make it better for his/her class?
9. To what extent did the teacher demonstrate having high expectations of students?
10. How well does the teacher listen?
11. To what extent does the teacher work on the development of problem-solving skills and strategies?
12. How would you rate the teacher on overall fidelity to the goals of the program?

C. Include specific observations pertaining to students which you consider to be significant. Observations that may shed some light on the following questions should be included:

1. What type of students are in the class?
2. To what extent are students familiar with the problem-solving skills that LCMP had identified for this particular level?
3. Are students working cooperatively?
4. Are students seeking help from others but staying at task when doing this?
5. Do students seek resources or references?
6. What type of questions are students asking? "What if?," "Why?," etc.
7. Do students bring problems from home?
8. Do students get upset with lack of direction?
9. To what extent are students willing to guess or make predictions?
10. To what extent do students want to be told rather than investigate for themselves?

Directions For Using Observation Chart

C. (continued)

11. Is there good rapport among students? (To what extent are students willing to listen to other students?)

12. Do the students transfer their problem-solving skills to problem-solving situations outside the classroom? Where?

D. Miscellaneous observations

Include a description of any out-of-the-ordinary circumstances such as a remodelling project, unique schedule, etc.

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